8. Linear Inequations

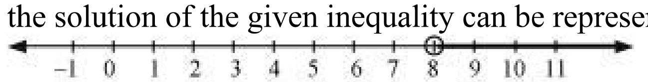
- Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an inequality.

Note: Inequalities involving '<' or '>'are strict inequalities whereas inequalities involving '≤' or '≥' are slack inequalities.

Example: $6 < 26$, $3 < z + 1 \leq 22$, $27 \geq s \geq 16$, $p + t > 100$

- Any **solution of an inequality in one variable** is a value of the variable that makes it a true statement.
- The set of numbers consisting of all the solutions of an inequality is known as the **solution set** of the inequality.
- The rules that need to be followed to solve an inequality are:
    - Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of the inequality.
    - Both sides of an inequality can be multiplied (or divided) with the same positive number. However, when both sides are multiplied or divided by a negative number, then the sign of the inequality is reversed.
- To represent $x \leq a$ (or $x \geq a$) on a number line, encircle the number a, and darken the line to the left (or the right) of a.

Example:
Show the graph of the solution of the inequality $5(x - 3) > 2x + 9$ on number line.
Solution:
$5(x - 3) > 2x + 9$
$\Rightarrow 5x - 15 > 2x + 9$
$\Rightarrow 5x - 15 - 2x > 2x + 9 - 2x$
$\Rightarrow 3x - 15 > 9$
$\Rightarrow 3x > 9 + 15$
$\Rightarrow 3x > 24$
$\Rightarrow x > 8$
Thus, the solution of the given inequality can be represented on the number line as shown below.



- The solution set might be taken from real numbers or whole numbers or integers or any other set of numbers. The set from which the values of the variables (involved in the inequation) are chosen is called the **replacement set**. We may take any set as the replacement set. For example, **N**, **Z**, $\{-4, -3, -2\}$ can be taken as the replacement set.

- **Linear inequalities in two variables and representing their solution graphically**

Rules for solving an inequality:

- Equal numbers may be added to or subtracted from both sides of an inequality without affecting the sign of the inequality.

- Both sides of an inequality can be multiplied with or divided by the same positive number. But when both sides are multiplied with or divided by a negative number, the sign of inequality is reversed.

Example 1: Solve $3\left(\frac{3}{5}x+4\right) \geq 2(x-3)$.

Solution:

$3\left(\frac{3}{5}x+4\right) \geq 2(x-3)$

$\Rightarrow 3\left(\frac{3x+20}{5}\right) \geq 2(x-3)$

$\Rightarrow 3(3x + 20) \geq 10(x - 3)$

$\Rightarrow 9x + 60 \geq 10x - 30$

$\Rightarrow 9x - 10x \geq -30 - 60$

$\Rightarrow -x \geq -90$

$\Rightarrow x \leq 90$

∴ The solution set of the given inequality is $(-\infty, 90]$.

The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.

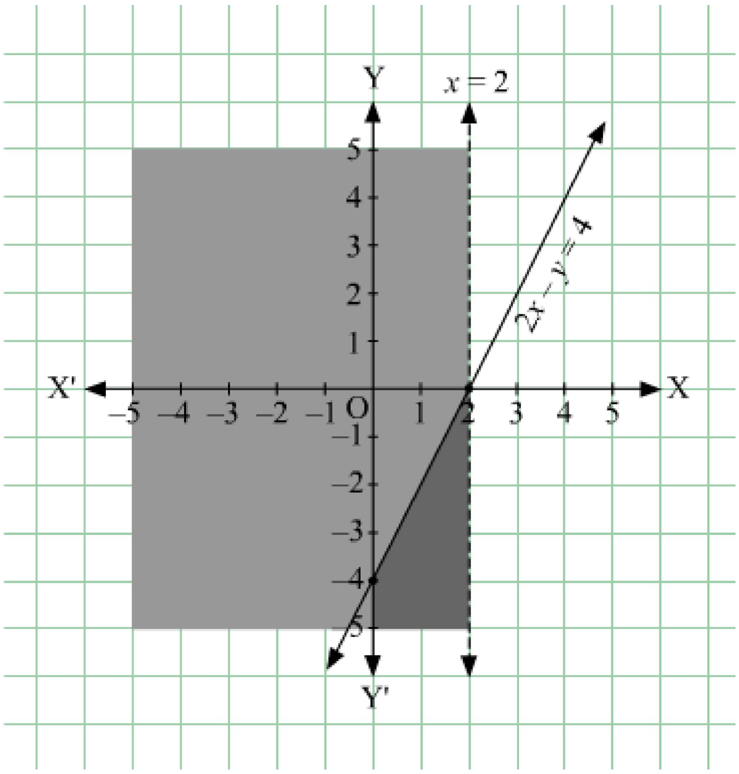
In order to identify the half plane represented by an inequality, it is sufficient to take any point (a, b) (not on the line) and check whether it satisfies the inequality or not. If it satisfies, then the inequality represents the half plane containing the point and we shade this region. If not, then the inequality represents the half plane which does not contain the point. For convenience, the point (0, 0) is preferred.

In an inequality of the type $ax + by \geq c$ or $ax + by \leq c$, the points on the line $ax + by = c$ are to be included in the solution region. So, we darken the line in the solution region.

In an inequality of the type $ax + by > c$ or $ax + by < c$, the points on the line $ax + by = c$ are not to be included in the solution region. So, we draw a broken or dotted line in the solution region.

Example 2: Solve the following system of linear inequalities graphically: $2x - y \leq 4$, $x < 2$
Solution: The given linear inequalities are
$2x - y \leq 4$ … (i)
$x < 2$ … (ii)

The graphs of the lines $2x - y = 4$ and $x = 2$ are drawn in the figure below.



Inequality (i) represents the region on the left of the line $2x - y = 4$ (including the line $2x - y = 4$). Inequality (ii) represents the region on the left of the line $x = 2$ (excluding the line $x = 2$).
Hence, the solution of the given system of linear inequalities is represented by the common shaded region, including the points on the line $2x - y = 4$.

Convex Set

- A set of points in a plane is said to be a convex set if the line segment joining any two points of the set lies completely within the set.
- A convex set can be bounded or unbounded.
- A convex set whose boundaries are formed by straight lines is called a polygonal convex set. It can be bounded or unbounded.
- A bounded polygonal convex set is also known as a **convex polygon**. All the vertices of a convex polygon point outwards and away from the interior of the shape. A line drawn through a convex polygon intersects the polygon exactly twice.

Feasible Solution

- Any point in a plane which satisfy a given set of inequations is said to be the feasible solution of the given set of inequations.

Steps to find the feasible solution of a system of inequations:

- Convert the given inequations into equations which represents the straight lines in the xy-plane.
- Put $x = 0$ in the equations to obtain the points where the straight lines meet the y-axis. Similarly, put $y = 0$ in the equations to obtain the points where the straight lines meet the x-axis.
- Join the points to obtain the graph of the straight lines obtained from the given inequations.
- Choose a convenient point, say (0, 0), which is not lying on the straight lines. Put the coordinates in the inequations. If the inequation is satisfied, then shade the portion of the plane that contains the chosen point, otherwise shade the portion of plane that does not contain the chosen point.
- The common shaded portion represents the feasible solution of the set of inequations.